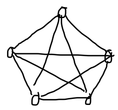
A. Kitaev – "A simple model of quantum holography: Part I"

04/07/15

Model:

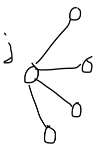


χ_j $\qquad \{\chi_j, \chi_k\} = \delta_{jk}$

$$H = \frac{1}{4!} \sum_{j,k,l,m} J_{jklm} \chi_j \chi_k \chi_l \chi_m \; ; \quad \frac{1}{3!} \sum_{k,l,m} \overline{J^2_{jklm}} = J^2$$

disordered

χ_j



$$\sum_{klm} J_{jklm} \chi_k \chi_l \chi_m \equiv O_j$$

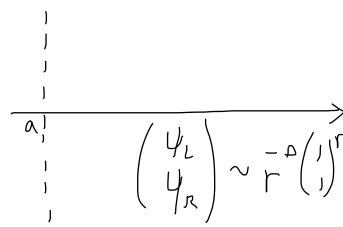
bath field

Parameters: $\left.\begin{array}{l} N \gg 1 \\ \lambda = \beta J \end{array}\right\}$ $\lambda \gg 1$ – holographic

Metric

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + \cancel{r^2 d\Omega^2}$$

where: $f(r) = \dfrac{r(r-a)}{R^2} \sim r(r-1)$



$$\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \sim r^{-\Delta} \begin{pmatrix} 1 \\ 1 \end{pmatrix}^{r} O(t)$$

Spectral density:

$$A_{O,O}(\omega) = \int e^{i\omega t} \{O(t), O(0)\}\, dt$$

$$A^{(hor)}_{\psi,\psi}(\omega) \sim \text{const.}$$

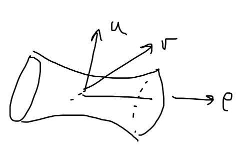
$$A_{O,O}(\omega) \sim \cosh\frac{\beta\omega}{2} \left| \Gamma\left(\frac{3}{4} - \frac{i\beta\omega}{2\pi}\right)\right|^2$$

Symmetries

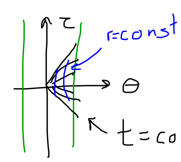
$$Z = e^{\frac{2\pi}{\beta} t}$$

$PSL(2,\mathbb{R})$

$\quad \cong SO(2,1)$



$$\left.\begin{array}{l} \rho = \tanh\theta \\ u = \rho\cos\tau \\ v = \rho\sin\tau \end{array}\right\}$$



$$ds^2 = \frac{-d\tau^2 + d\theta^2}{\cos^2\theta}$$

$$Z \to (1+\xi) Z$$
$$Z^{-1} \to Z^{-1} + \xi$$

Correlation Functions

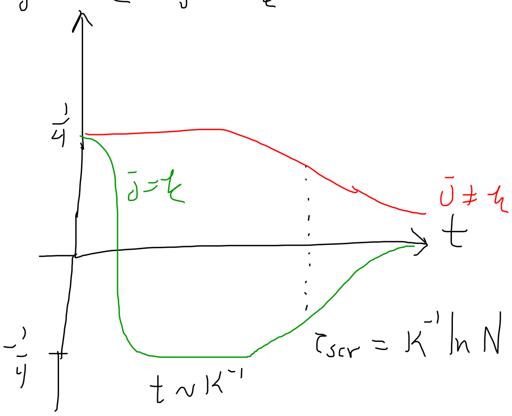
$$\langle D(t) C(0) B(t) A(0) \rangle$$

$$|\Psi\rangle = Z^{-\frac{1}{2}} \sum_n e^{-\frac{\beta E_n}{2}} |n, n\rangle \; ; \quad \sqrt{\rho} = Z^{-\frac{1}{2}} \sum_n e^{-\frac{\beta E_n}{2}} |n\rangle\langle n|$$

$$H\sqrt{\rho} - \sqrt{\rho} H \iff (H \otimes I - I \otimes H^{+}) |\Psi\rangle = 0$$

$$\langle \Psi | (U(0)^\dagger \otimes I)(X(t) \otimes Y(t))(U(0) \otimes I) | \Psi \rangle$$

$$\langle \chi_j(t)\chi_k(0)\chi_j(t)\chi_k(0)\rangle$$



$K = \frac{2\pi}{\beta}$ if $\lambda >> 1$ (strong coupling)

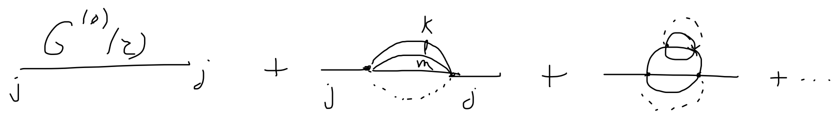
$K \sim J$ if $\lambda << 1$

$$\langle D(t)C(0)B(t)A(0)\rangle - \langle CA\rangle\langle DB\rangle \sim \frac{1}{N}e^{Kt} \quad ; \quad K^{-1} << t < K^{-1}\ln N$$

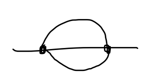
Calculations in the spin model

$$G_{jj} = -\langle T\chi_j(\tau)\chi_j(0)\rangle$$

Free mode: $G^{(0)}(\tau) = -\frac{1}{2}\operatorname{sgn}\tau \quad ; \quad S = \int\left[\frac{i}{2}\chi_j\dot{\chi}_j - H\right]dt$

$G_{jj}(\tau) =$ 

Dyson Equation: $G(i\omega_n)^{-1} = i\omega_n - \Sigma(i\omega_n) \quad ; \quad \omega_n = \frac{2\pi}{\beta}(n+\frac{1}{2})$

$\Sigma(\tau) = J^2 G(\tau)^3$ 

$G(+0) = -\frac{1}{2}, \quad G(-0) = +\frac{1}{2}$

$\beta^{-1} << \omega << J$: solution $G(\omega) = a\,\frac{1-i}{\sqrt{2}}\,\omega^{-\frac{1}{2}}$ for $\mathrm{Im}(\omega) > 0$

$$a = \left(\frac{1}{J}\right)^{-\frac{1}{4}}$$

@ strong coupling: ignore $i\omega_n$: $G\Sigma = 1$

$$\begin{cases} \int G(\tau_1,\tau_2)\Sigma(\tau_2,\tau_3)\,d\tau_2 = \delta(\tau_1-\tau_3) \\ \Sigma(\tau_1,\tau_2) = J^2 G(\tau_1,\tau_2)^3 \end{cases}$$

$$\left.\begin{aligned} G(\tau_1,\tau_2) &\to G(f(\tau_1), f(\tau_2)) \cdot f'(\tau_1)^{1-\Delta} f'(\tau_2)^{1-\Delta} \\ \Sigma(\tau_1,\tau_2) &\to \Sigma(f(\tau_1), f(\tau_2))\, f'(\tau_1)^{\Delta} f'(\tau_2)^{\Delta} \end{aligned}\right\} \Delta = \frac{3}{4}$$

$$f(\tau) = e^{\frac{2\pi i \tau}{\beta}} \;;\; G(\tau_1,\tau_2) = \tilde{G}(f(\tau_1), f(\tau_2))\, f'(\tau_1)^{1-\Delta} f'(\tau_2)^{1-\Delta}$$

$$\tilde{G}(z_1,z_2) = C(z_1-z_2)^{2(\Delta-1)}$$

$$G(\tau_1,\tau_2) = \frac{-a}{\sqrt{2\beta}}\left(\sin\frac{\pi(\tau_1-\tau_2)}{\beta}\right)^{-\frac{1}{2}}$$