1. Universal part of entanglement entropy (Σ = entangling surface).

- 1+1 CFT: $S \sim \frac{c}{6}\,\text{Volume}(\Sigma)\,\log\frac{L}{\epsilon}$

3+1 CFT: $S \sim C_{\text{eff}}\,\text{Volume}(\Sigma)\,\frac{1}{\epsilon^2}$

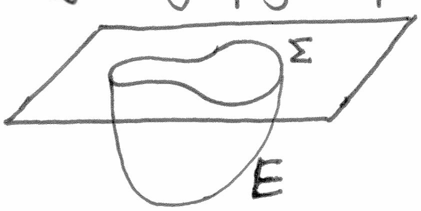
$$- \log\frac{L}{\epsilon}\left[\frac{a}{2\pi}\int_\Sigma R_\Sigma + \frac{c}{2\pi}\int_\Sigma\left(\text{Tr}K^2 - \tfrac{1}{2}(\text{Tr}K)^2 - C^{ab}{}_{ab}\right)\right]$$

bending of embedded surface

Solodukhin '08.

2+1 CFT: $S \sim C_{\text{eff}}\,\text{Volume}(\Sigma)\,\frac{1}{\epsilon} - \gamma + \cdots$

Holography reproduces 1+1 & 3+1:



$$S = \frac{\text{Area}(E)}{4G_N} \qquad (EG)$$

Need near boundary expansion of E, not full solution.

(Trick: use PBH transformations to constrain expansion: infinitesimal diff that preserves FG gauge)

bending $\sim -\frac{L_{AdS}^2}{2(d-2)}K$

Einstein gravity fully produces except that $a=c$.
(go beyond that?)

Can also calculate odd dimensional cases holographically.

Reyni entropy: (more information about entanglement spectrum).

1+1: $S_n \sim \frac{c}{12}\left(1+\frac{1}{n}\right)\text{Vol}_\Sigma \log\frac{L}{\epsilon}$

3+1: $S_n \sim$ Vol law $- \log \frac{L}{\epsilon} \left[\frac{f_a(n)}{2\pi} \int_\Sigma R_\Sigma + \frac{f_b(n)}{2\pi} \int_\Sigma \left(\mathrm{Tr} K^2 - \frac{1}{2} (\mathrm{Tr} K)^2 \right) + \frac{f_c(n)}{2\pi} \int_\Sigma C^{ab}{}_{ab} \right]$

Fursaev 12

$f_{a,b,c}(n=1) = a, c, c.$

shown generally: $f_c(n) = \frac{n}{n-1} \left[a - f_a(n) - (n-1) f_a'(n) \right]$ Lewkowycz & Perlmutter

conjecture: $f_b(n) = f_c(n)$ [Lee, McGough, Safdi]

proof (holographically).

2. Higher derivative gravity

① ☺ was produced in Einstein gravity ($a = c$)

② Large λ limit, back away? (will deal with large N later).

$$L = \frac{R}{16\pi G_N} + g_1 R^2 + g_2 R_{\mu\nu} R^{\mu\nu} + g_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ldots$$

EH action [derivative expansion].

ⓐ Now $a \neq c$.

ⓑ I derived a general formula for entropy in these theories:

$$S = \int_E \underbrace{\frac{\partial L}{\partial R_{z\bar{z}z\bar{z}}}}_{\text{Wald's formula}} + \frac{\partial^2 L}{\partial R_{ziz j} \partial R_{\bar{z}k\bar{z}l}} \underbrace{K_{zij} K_{\bar{z}kl}}_{\text{extrinsic curvature}}$$

$\boxed{D13}$

E codim-2

$D \to D-2$

Easy check: $L = R \to S = \frac{\text{area}}{4G_N}$.

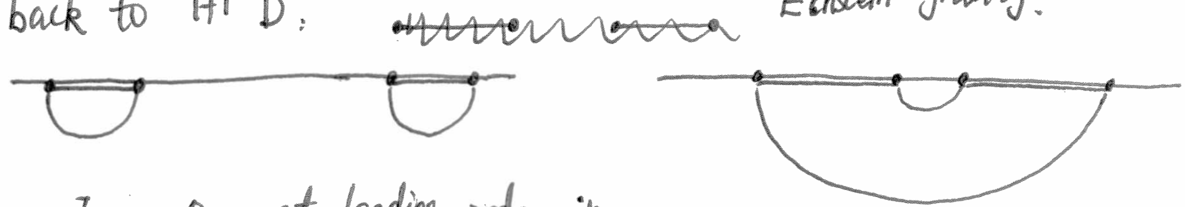
ⓒ With this prescription, one can fully verify ☺.

$$S = \frac{\text{area}}{4G_N} - 4\pi \int \left[2g_1 R + g_2 \left(R^a{}_a - \frac{1}{2} K_a K^a \right) + 2g_3 \left(R^{ab}{}_{ab} - K_{aij} K^{aij} \right) \right] + \ldots$$

Fursaev, Patrushev, Solodukhin '13. ...

3. Phase transition, $\frac{1}{N}$ corrections,

back to 1+1 D:



$I_{AB} = 0$ at leading order in large N.

$I_{AB} = \mathcal{O}(N^2)$.

$$\frac{L_{AdS}^{d-1}}{G_N} = \left(\frac{L_{AdS}}{l_{planck}}\right)^{d-1} = N^2$$

Can derive (without using RT): using AdS/CFT — Faulkner 13.
using large-c & Δ_{gap} — Hartman 13.

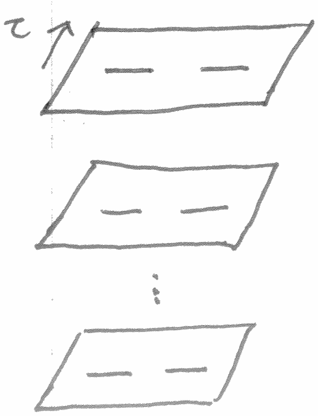
$I_{AB} > 0$ so in first phase $I_{AB} = 0N^2 + \mathcal{O}(1) + \cdots$

$\uparrow$ 1/N correction. (quantum corrections) in the bulk.

① These $\frac{1}{N}$ corrections are obtained by bulk entanglement across E (FLM; Netta Engelhardt, Wall)

② 1+1 D; explicitly calculate with AdS/CFT.

$$S_n = -\frac{1}{n-1} \log \mathrm{Tr}\rho^n = -\frac{1}{n-1} \log \frac{Z_n}{Z_1^n}$$



genus n-1

① Calculate Z_n using AdS/CFT: $(Z_{CFT} = Z_{grav})$.

② Need to construct gravity dual of genus n-1 surface "fill in the solid handlebody" subject to Einstein eqn.

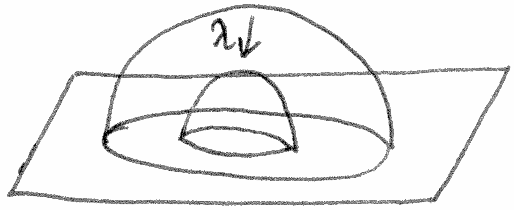
③ Take simple case of pure gravity (no other fields).
"simplest gravitational theory"

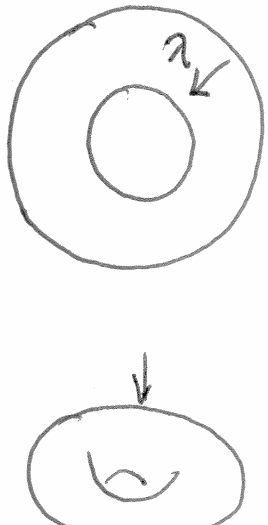
Only sols to EE (with neg c.c.) in 3-dim must be locally AdS_3.

So full solution is AdS_3/Γ, $\Gamma \subset \underset{\text{isometry of } AdS_3.}{PSL(2,\mathbb{C})}$

This works both at the boundary and in the bulk.

ex $g=1$:





$\Gamma = \langle \lambda \rangle$

$(x_i, z) \rightarrow \lambda (x_i, z)$.

④ ~~Construct~~ We want to find Γ s.t. the surface $M_n = \partial AdS_3/\Gamma$
"Schottky uniformization"
Solve an 2nd order ODE with monodromy conditions to find the right Γ

⑤ ~~Classically~~ At leading order in $\frac{1}{N}$, gives RT (classical gravity).

⑥ "Quantum order":

$$I_{\text{one-loop}} = \frac{x^4}{630} + \frac{2x^5}{693} + \frac{15x^6}{4004} + \frac{x^7}{234} + \frac{167x^8}{36936} + \mathcal{O}(x^9)$$

$$x = \frac{(z_2 - z_1)(z_4 - z_3)}{(z_3 - z_1)(z_4 - z_3)}.$$

Calabrese, John Cardy, Eric Tonni '11 : $S = -N \left(\frac{x}{4}\right)^{2h} \frac{\sqrt{\pi}}{4} \frac{\Gamma(2h+1)}{\Gamma(2h+\frac{3}{2})}$ + ...

$h=2$, stress tensor, mult. $N=2$